COMPRESSED TRANSIENT ANALYSIS SPEEDS UP THE PERIODIC STEADY STATE ANALYSIS OF NONLINEAR MICROWAVE CIRCUITS

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ABSTRACT

Harmonic Balance is nowadays the most efficient method for the periodic steady state analysis of microwave circuits. Unfortunately this method cannot conveniently handle realistic wide band input signals, because its computation time grows rapidly with the number of harmonics. A new method is proposed which resolves the above mentioned limitation of the Harmonic Balance. A substantial computation time saving is obtained with respect to the Harmonic Balance.

I - INTRODUCTION

Harmonic Balance (HB) technique [1] is today the most widely used method for calculating the steady state response of nonlinear microwave circuits. This is based on the solution of the circuit equation in the frequency domain, over the entire period of the steady state regime. The use of DFT in order to compute the state variable harmonics to be balanced, produces however a system of tightly coupled equations. For this reason, in general, only Newton's method is successful for the solution of the HB equation [2] and the computation cost varies as N^3 where N is the number of significant harmonics of the steady state regime. When N is large, this cost becomes prohibitive. Hence HB technique is only suitable for circuit analysis under monochromatic excitation. Realistic wide band input signals like a square wave cannot be carried conveniently.

Alternatively, Time domain "brute force" integration method exhibits a series of loosely coupled equations that have to be solved from the time origin till the transient has died out. This method is more efficient than HB if the transient disappears after a few periods. However, the transient of microwave circuits is usually many order larger than the period of the microwave signal. Therefore the computation cost for finding the steady state is prohibitive.

This paper presents a new and general purpose technique termed "*Compressed Transient*" for formulating circuit steady state equations. This method produces a time domain equation system which exhibits an artificial and short transient, so that the desired steady state response is obtained after a few periods, irrespective of the input signal period.

As a consequence, a substantial computation time saving of a factor 10 and more is obtained, if compared to the HB technique.

In the following, we will present the Compressed Transient (CT) technique and a wide-band amplifier example which shows the efficiency and effectiveness of the new method. In the conclusion, the extension of the presented method to multitone analysis is indicated.

II - THE COMPRESSED TRANSIENT TECHNIQUE

II.1 Formulation

In the HB technique, the circuit is generally divided in a linear and a nonlinear subcircuits. The equation to be solved is discrete in the frequency domain and has the general form below.

$$\begin{cases} X_k = A_1(k\omega_0)Y_k + A_2(k\omega_0)G_k \\ -N \le k \le N \\ y(t) = f(x(t)) \end{cases} \quad (1)$$

where X_k, Y_k and G_k are respectively the kth frequency components of the state variables $x(t)$, the electrical variables at the nonlinear subcircuit ports $y(t)$ and the driving sources $g(t)$. ω_0 is the fundamental frequency of the steady state regime, $y(t) = f(x(t))$ represents the intrinsic characteristic of the nonlinear subcircuit, and $A_1(\omega)$, $A_2(\omega)$ are

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the transfert functions characterizing the linear subcircuit.

An important consideration from equation (1) is to be kept in mind. The steady state response of the circuit depends solely on the value taken by $A_1(\omega)$ and $A_2(\omega)$ at DC and multiples of the fundamental ω_0.

Let us consider the auxiliary equation below, which in contrast to (1), is continuous in the frequency domain.

$$\begin{cases} X(\omega) = \hat{A}_1(\omega)Y(\omega) + \hat{A}_2(\omega)G(\omega) \\ -\infty < \omega < \infty \\ y(t) = f(x(t)) \end{cases} \qquad (2)$$

If we satisfy the following condition,

$$\begin{cases} \hat{A}_1(k\omega_0) = A_1(k\omega_0) \\ \hat{A}_2(k\omega_0) = A_2(k\omega_0), \qquad -N \le k \le N \end{cases} \qquad (3)$$

then equation (2) yields a steady state solution identical to that produced by (1). There is therefore a family of continuous transfert functions $\hat{A}_1(\omega)$ and $\hat{A}_2(\omega)$ that exhibit the same steady state regime as (1).

Applying the inverse Fourier transform to (2), we find the following convolution equation

$$\begin{cases} x(t) = \int_0^{\tau_{max}} \hat{a}_1(\tau)y(t-\tau)d\tau + \int_0^{\tau_{max}} \hat{a}_2(\tau)g(t-\tau)d\tau \\ y(t) = f(x(t)) \end{cases} \qquad (4)$$

where τ_{max} is the maximum duration of the impulse responses $\hat{a}_1(t)$ and $\hat{a}_2(t)$.

To obtain the steady state solution of (2), equation (4) may be integrated from $t = 0$ until the transient dies out at time $t = t_\infty$. If function $f()$ is a linear operator, the transient duration $t_\infty = \tau_{max}$. In nonlinear circuits however, $f()$ is nonlinear and the transient duration t_∞ is larger than τ_{max} and depends on the decay of the impulse response $\hat{a}_1(t)$. The more the impulse response will be confined to the time origin, the shorter the transient duration will be.

The philosophy of the compressed transient technique is therefore to find the auxiliary transfert functions $\hat{A}_1(\omega)$ and $\hat{A}_2(\omega)$ or equivalently the impulse responses $\hat{a}_1(t)$ and $\hat{a}_2(t)$ satisfying equation (3) and exhibiting the shortest transient possible. As $\hat{A}_1(\omega)$ and $\hat{A}_2(\omega)$ satisfy (3), the resulting steady state response is identical to the one obtained by Harmonic Balance.

For reasons beyond the scope of this summary, we have chosen an impulse response of the general form,

$$\hat{a}(t) = p(t) \sum_{k=-N}^{N} B_k e^{jk\omega_0 t} \qquad (5)$$

where $p(t)$ is an exponential decay confining function.

$$p(t) = \frac{1}{T_0} e^{-\frac{\alpha t}{T_0}} rect\left(\frac{t - T_0/2}{T_0}\right) \qquad (6)$$

T_0 is the period of the steady state response and α a positive argument controling the decay of the impulse response.

Taking the Fourier transform of (5) and introducing into (3), we find the de-embedding equation for the complex coefficients B_k.

For illustration, if we consider a simple RC transfert function

$$\begin{cases} A(k\omega_0) = \dfrac{1}{1 + jk\omega_0 RC} \\ R = 10\Omega,\ C = 10pF, \\ \omega_0 = 2\pi \times 10GHz \text{ and } N = 10 \end{cases} \qquad (7)$$

Fig 1 gives the resulting scaled impulse responses $\frac{\hat{a}(t)}{\hat{a}(0)}$ for various values of the decay factor α. One may observe that when α is increased, the impulse response is well confined to the time origin.

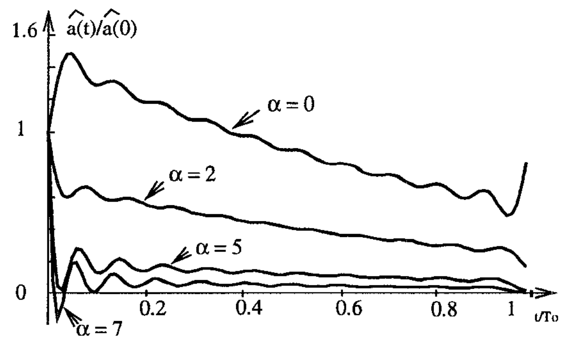

Figure 1 : Auxiliary impulse responses $\hat{a}(t)$ for varying α

II.2 Discretizing the convolution integral

Once the impulse responses $\hat{a}_1(t)$ and $\hat{a}_2(t)$ are computed, we can discretize the convolution integral in (4), with a time step $\Delta t \leq \frac{T_0}{2N+1}$, in order to carry a transient analysis. The discrete convolution equation can then be easily put in the form

$$\begin{cases} x(n) = \sum_{m=0}^{2N} y(n-m)a_1(m) + \sum_{m=0}^{2N} g(n-m)a_2(m) \\ y(n) = f(x(n)) \end{cases} \quad (8)$$

where $a_1(n)$ and $a_2(n)$ are the discrete images of $\hat{a}_1(t)$ and $\hat{a}_2(t)$.

Equation (8) may be solved from $n = 0$ until the transient dies out. For a decay factor $\alpha \geq 5$, this usually occurs very rapidly in a few periods, typically less than 10.

We have found that the convenient decay factor α for achieving a good impulse response confining is the range 5 to 10. $\alpha << 5$ does not produce enough impulse confining to guarantee a short transient. On the orther hand, $\alpha >> 10$ leads to numerical instabilities in computing the coefficients B_k as N increases.

Here it is worth noting that when $\alpha = 0$, no impulse confining is done, and $a_1(n)$ is just the DFT of the discrete transfert function $A_1(k\omega_0)$. In this case, eq (8) is equivalent to the waveform-balance or convolution equation already proposed by many authors [3-5].

III - SIMULATION RESULTS

We have considered the analysis of a four stage (4x300μm MESFETs) MMIC distributed amplifier recently manufactured at Texas Instruments foundry [6].

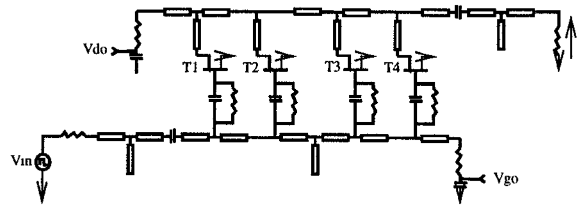

Figure 2 : 2-18 GHz MMIC Distributed Power Amplifier

This amplifier (Fig. 2) is a medium power amplifier in the 2-18Ghz band, exhibiting a 14% added power efficiency and a 27% drain efficiency for a 340mW/mm power density at 1dB gain compression. The analysis of the amplifier is carried with a square wave input signal of 2GHz fundamental frequency, at 2dB compression. The number of harmonics considered for analysis is swept from 5 to 80. Fig.3 gives the plot of simulation CPU time versus the number of harmonics for both HB and CT techniques on a HP9000 series 700. One may observe, as expected the substantial saving in CPU time obtained with the CT method. This saving goes increasing with the number of harmonics, and is about a factor 40 for N above 20.

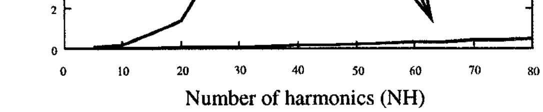

Figure 3 : CPU Time versus the number of harmonics for Harmonic Balance and Compressed Transient methods

It is found from the results that a number of harmonics of at least 50 is necessary to conveniently represent a 1% rise time of the square wave. Fig.4 shows the input and output voltage waveforms for both the HB and the CT methods. The agreement is good. The computation time is 20 minutes for HB and 10 seconds for CT.

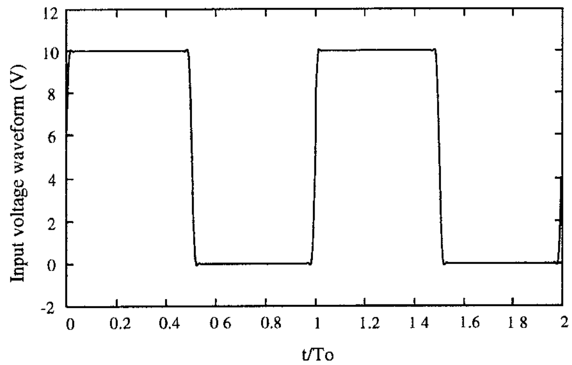

Figure 4a : Input voltage waveform

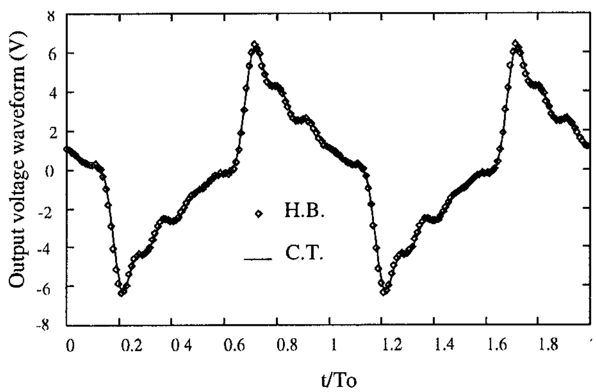

Figure 4b : Output voltage waveform obtained by Harmonic Balance and Compressed Transient for 50 harmonics

IV - CONCLUSION

A new method termed "*Compressed transient*" has been presented for the steady state analysis of nonlinear microwave circuits. This method, like the popular Harmonic Balance, allows to carry efficiently circuit analysis with sinusoidal as well as realistic wide-band input signals. A computation time saving of a factor more than 10 is obtained with respect to the Harmonic Balance. For paper length reasons, the principle of the compressed transient was presented only for the case of periodic steady state. This principle can however be easily extended to 2 or 3 tones analysis using a multidimensionnal Fourier transform representation of the signals.

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